Unified International Mathematics Olympiad

UIMO SAMPLE QUESTIONS

CLASS - 11

MATHEMATICS - 1

01. **A person appears for an examination in which there are four papers with a maximum of '*m*' marks from each paper. Find the number of ways in which one can get 2m marks.**

(A) ${}^{2m+3}C_3$ (B) $\frac{1}{3}(m+1)(2m^2+4m+1)$

(C) $2m+3$ (D) $\frac{1}{3}(m+1)(2m^2+4m+3)$

02. **What is the degree of the polynomial in x for the expression given below ?**

$$\frac{1}{\sqrt{4x+1}}\left\{\left(\frac{1+\sqrt{4x+1}}{2}\right)^7-\left(\frac{1-\sqrt{4x+1}}{2}\right)^7\right\}$$

(A) 7 (B) 5 (C) 4 (D) 3

03. **If $i^2=-1$, then the sum $\cos45°+i.\cos135°+\ldots.+i^n\cos(45+90n)°+\ldots+i^{40}\cos3645°$ equals _____**

(A) $\frac{\sqrt{2}}{2}$ (B) $-10i\sqrt{2}$ (C) $i\frac{\sqrt{2}}{2}(21-20i)$ (D) $\frac{21\sqrt{2}}{2}$

04. **If $x^2-(1-2i)x=\left(\frac{1}{2}+i\right)$, then the complete solution is:**

(A) $\left\{\frac{1\pm2i}{2}\right\}$ (B) $\left\{\frac{1\pm3i}{3}\right\}$ (C) $\left\{\frac{1+i}{i},\frac{1+3i}{2}\right\}$ (D) $\left\{\frac{1-i}{2},\frac{1-3i}{2}\right\}$

05. **If $y=x^{\sin x}+(\sin x)^x$, then $\frac{dy}{dx}$ = _____**

(A) $x^{\sin x}\left(\frac{x}{\sin x}+\cos x.\log x\right)+(\sin x)^x(x\cot x-\log\sin x)$

(B) $x^{\sin x}\left(\frac{\sin x}{x}+\cos x.\log x\right)+(\sin x)^x(x\cot x+\log\sin x)$

(C) $x^{\sin x}(x\cdot\operatorname{cosec}x-\cos x.\log x)+(\sin x)^x(x.\cot x+\log\cos x)$

(D) $x^{\sin x}(x\cdot\cos x-\sin x\cdot\log x)+(\sin x)^x(x\cdot\tan x+\log\sin x)$

MATHEMATICS - 2

01. Two points A and B in a plane are related if OA = OB, where O is a fixed point. This relation is

(A) reflexive and symmetric but not transitive. (B) an equivalence relation.

(C) reflexive and symmetric. (D) reflexive and transitive.

02. Which of the following is/are not an empty set ?

(A) $\{x|x$ is a real number and $x^2 - 1 = 0\}$ (B) $\{x|x$ is a real number and $x^2 + 1 = 0\}$

(C) $\{x|x$ is a real number and $x^2 - 9 = 0\}$ (D) $\{x|x$ is a real number and $x^2 = x + 2\}$

03. Which of the following is true about the graph of the inequations $x \geq 0$, $y \geq 0$, $3x + 4y \leq 12$?

(A) Exterior of a triangle. (B) Triangular region including the points on the sides.

(C) Lies in the first quadrant. (D) It is a rectangle.

04. Which of the following is true ?

(A) ${}^nC_r = {}^nC_{n-r}$ (B) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ (C) ${}^nP_r = r!\ {}^nC_r$ (D) ${}^nC_r + {}^nC_{r-1} = {}^nC_{r+1}$

05. If A_1 and A_2 are two arithmetic means between two numbers a and b, then $(2A_1 - A_2)(2A_2 - A_1)$ is not equal to ?

(A) a + b (B) ab (C) $\frac{ab}{a+b}$ (D) $\frac{2ab}{a+b}$

REASONING

01. In a certain code, KAVERI is written as VAKIRE. How is MYSORE written in that code ?

(A) EROSYM (B) SYMERO (C) SEYMRO (D) SMYERP

02. Find out how many such pairs of letters are there in the given word each of which has as many letters between them in the word as in the English alphabet.

PRISON

(A) More than three (B) One (C) Two (D) Three

03. Choose the odd one from the following.

(A) Dynamics (B) Mechanics (C) Physics (D) Optics

04. Find out the wrong term in the following number series.

25, 36, 49, 81, 121, 169, 225

(A) 36 (B) 49 (C) 169 (D) 225

05. A rat runs 20 m towards East and turns to right, runs 10 m and turns to right, runs 9 m and again turns to left, runs 5 m and then turns to left, runs 12 m and finally turns to left and runs 6 m. Now, which direction is the rat facing ?

(A) East (B) West (C) North (D) South

CRITICAL THINKING

01. **What is the angle between the two hands of a clock when the time shown by the clock is 6.30 p.m. ?**

(A) 0° (B) 50° (C) 30° (D) 15°

02. **Choose the option which is the upper half of the word.**

(A) 

(B) 

(C) 

(D) 

03. **Many years ago, 1st March fell on Firday. On which day of the week was 15th April in the same year ?**

(A) Monday (B) Tuesday

(C) Thursday (D) Friday

04. **In a class there are seven students (including boys and girls) P, Q, R, S, T, U, and V. They sit on three benches I, II and III such that at least two students on each bench and at least one girl on each bench. R who is a girl student, does not sit with P, T and S. U the boy student sits with only Q. P sits on the bench I with his best friends. V sits on the bench III. T is the brother of R. Which of the following is the group of girls ?**

(A) QPR (B) QUR (C) QRS (D) RSU

05. **During the past year, John saw more movies than Suman. Suman saw fewer movies than David. David saw more movies than John. If the first two statements are true, the third statement is**

(A) true (B) false

(C) uncertain (D) data inadequate

KEY & SOLUTION

MATHEMATICS - 1

01. **(D)** The required number

$= \text{coeff.of } x^{2m} \text{ in } (x^0 + x^1 + ... + x^m)^4$

$= \text{coeff.of } x^{2m} \text{ in } \left(\frac{1-x^{m+1}}{1-x}\right)^4$

$= \text{coeff.of } x^{2m} \text{ in } (1-x^{m+1})^4 (1-x)^{-4}$

$= \text{coeff.of } x^{2m} \text{ in } (1-4x^{m+1} + 6x^{2m+2} + ...)$

$\left(1 + 4x + ... + \frac{(r+1)(r+2)(r+3)}{3!}x^r + ...\right)$

$= \frac{(2m+1)(2m+2)(2m+3)}{6} - 4m\frac{(m+1)(m+2)}{6}$

$= \frac{(m+1)(2m^2+4m+3)}{3}$

02. **(D)** $\frac{1}{\sqrt{4x+1}}\left\{\left(\frac{1+\sqrt{4x+1}}{2}\right)^7 - \left(\frac{1-\sqrt{4x+1}}{2}\right)^7\right\}$

$= \frac{1}{2^7\sqrt{4x+1}}\{2\{^7C_1\sqrt{4x+1} + {}^7C_3(\sqrt{4x+1})^3$

$+ {}^7C_5(\sqrt{4x+1})^5 + {}^7C_7(\sqrt{4x+1})^7\}\}$

$= \frac{1}{2^6}\{^7C_1 + {}^7C_3(4x+1) + {}^7C_5(4x+1)^2 + {}^7C_7(4x+1)^3\}$

Clearly, it is a polynomial of degree 3.

03. **(C)** Since, $i^{n+2}\cos[45 + 90(n+2)]^\circ$

$= -i^n[-\cos(45 + 90n)^\circ]$

$= i^n[\cos(45 + 90n)^\circ]$

Every other term has the same value. The first is $\frac{\sqrt{2}}{2}$, and these are 2 terms with this value (n = 0, 2, 4, ..., 40).

The second term is i.cos 135° = $-\frac{i\sqrt{2}}{2}$ and there are 20 terms with this value (n = 1, 3, ..., 39)

Thus the sum is $\frac{i\sqrt{2}}{2}(21 - 20i)$

04. **(D)** $x^2 - (1 - 2i)x = \left(\frac{1}{2} + i\right)$

$\Rightarrow 2x^2 + (-2 + 4i)x - 1 - 2i = 0$

$x = \frac{-(-2+4i) \pm \sqrt{(-2+4i)^2 - 4(2)(-1-2i)}}{4}$

$= \frac{2 - 4i \pm \sqrt{-4}}{4} = \frac{2 - 4i \pm 2i}{4}$

$\Rightarrow x = \frac{1-i}{2}$ or $\frac{1-3i}{2}$

05. **(B)** $\frac{dy}{dx} = x^{\sin x}\left\{\frac{1}{x}\sin x + \log x.\cos x\right\}$

$+ (\sin x)^x \left\{\frac{\cos x}{\sin x}.x + \log(\sin x)\right\}$

$= x^{\sin x}\left(\frac{\sin x}{x} + \log x.\cos x\right) + (\sin x)^x$

[x.cotx + log(sinx)]

MATHEMATICS - 2

01. **(B,C,D)** It is an equivalence relation.

02. **(A,C,D)** $x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1 \in R$

$x^2 + 1 = 0 \Rightarrow x^2 = -1 \Rightarrow x = \pm \sqrt{1} \notin R$

$x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3 \in R$

$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = 2$ (or) $-1 \in R$

03. **(B,C)** It is a triangular region including the points on the sides which lies in first quadrant.

04. **(A,B,C)** (A) $^nC_{n-r} = \frac{n!}{(n-(n-r))!(n-r)!} = \frac{n!}{(n-r)!r!} = {}^nC_r$

(B) $^nC_r + {}^nC_{r-1} = \frac{n!}{(n-r)!r!} + \frac{n!}{[n-(r-1)]!(r-1)!}$

$= \frac{n!}{(n-r)!r!} + \frac{n!}{[n-r-1]!(r-1)!}$

$= \frac{n!}{(n-r)!(r-1)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$

$= \frac{n!}{(n-r)!(r-1)!}\left[\frac{1-r+1+r}{(r)(n-r+1)}\right]$

$$= \frac{(n+1)(n!)}{(n-r+1)(n-r)!r(r-1)!}$$

$$= \frac{(n+1)!}{(n+1-r)!r!} = {}^{(n+1)}C_r$$

$$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \neq {}^nC_{r+1}$$

(C) $${}^nP_r = \frac{n!}{(n-r)!} \times \frac{r!}{r!} = \frac{n!}{(n-r)!r!} \Rightarrow {}^nC_r \times r!$$

05. **(A,C,D)** Given a, A_1, A_2, b are in AP

$\therefore A_2 = b - d$

$\therefore d = A_1 - a = A_2 - A_1 = b - A_2$

$\therefore A_1 - a = A_2 - A_1 \Rightarrow 2A_1 - A_2 = a$

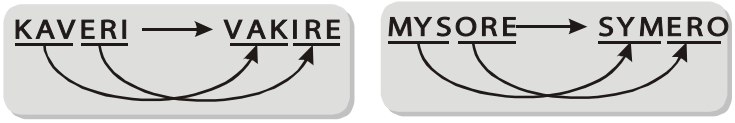
$A_2 - A_1 = b - A_2 \Rightarrow 2A_2 - A_1 = b$

$\therefore (2A_1 - A_2)(2A_2 - A_1) = ab$

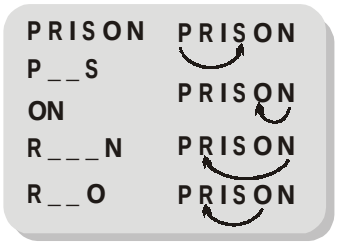
$$\therefore (2A_1 - A_2)(2A_2 - A_1) \neq \frac{ab}{a+b} \neq (a+b) \neq \frac{2ab}{a+b}$$

REASONING

01. **(B)**



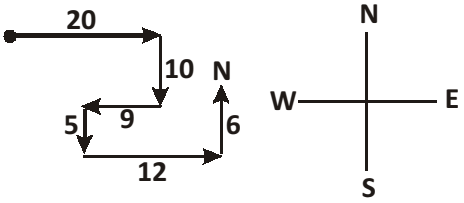
02. **(A)**



03. **(C)** Dynamics, Mechanics and optics are parts of physics.

04. **(A)** Except 36, all are the squares of odd numbers.

05. **(C)**



CRITICAL THINKING

01. **(D)** $6 \times 30° - 30 \times \frac{11}{2}$

$180 - 165 = 15°$

02. **(D)** KNOWLEDGE

03. **(A)** 1st March – Friday

30 days in march + 15 days in April

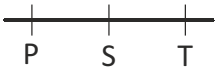
$\frac{45}{7}$ days = 3 odd days

$\therefore$ 15th April is Monday

04. **(C)**

I						III
P	Q	R	S	T	U	V
		girl			boy	

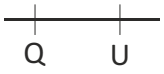
I



T is brother of R, T is boy. P sits on the bench I with his frids, P is boy

$\therefore$ S is girl

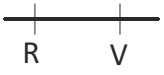
II



U is the boy student sits with only Q

$\therefore$ Q is girl

III



R does not sit with P, S, T

$\therefore$ R is in 3rd bench

R is girl

05. **(C)** If the first two statements are true, the third statement is uncertain.